Finite Difference Model of Sea Ice

Rachel Myers, André Shannon

Abstract

Our goal was to create a model of varying temperatures throughout the ice in the hopes of being able to use it to predict the depth of the ice using surface temperatures. This is a difficult task due to the layer of “slush” found between solid ice and seawater. This layer is a mixture of ice and water, making it difficult to map the thermal conductivity in that area. We used the formula P/A=k(dT/dz) as the basis for our model, with P representing power, A representing area, k representing thermal conductivity, dT representing the difference in temperature, and dz representing the thickness. We experimented with different approaches using Microsoft Excel. Our first model used the flux of heat in and out of the ice to plot the relationship between temperature and various heat coefficients. We then used the relationship found from this model to develop a way to alter k-values to represent the slush layer. We designed our model to represent estimated temperatures at different internal points in the vertical direction using the k-value of water right before it starts to freeze, the k-value of ice, and estimates of the ice temperature. We started with an extremely simplified linear model of the temperatures. The resultant k-values seemed to follow a logarithmic pattern. After that, we used the iterative functions of Excel to design a model composed of twelve temperature layers and the data that was collected on ice surface temperatures during our research.

Introduction

- Ice data collected in Barrow, Alaska
- The ice had been freezing for several months at the time the data were collected
- About 1 meter thick and extends from shore to a few kilometers out into the ocean
- Surface of the ice is about -20° Celsius (about -4° Fahrenheit)
- Ice composition changes vertically downward from solid ice to a mixture of ice and water (we call this the “slush layer”)

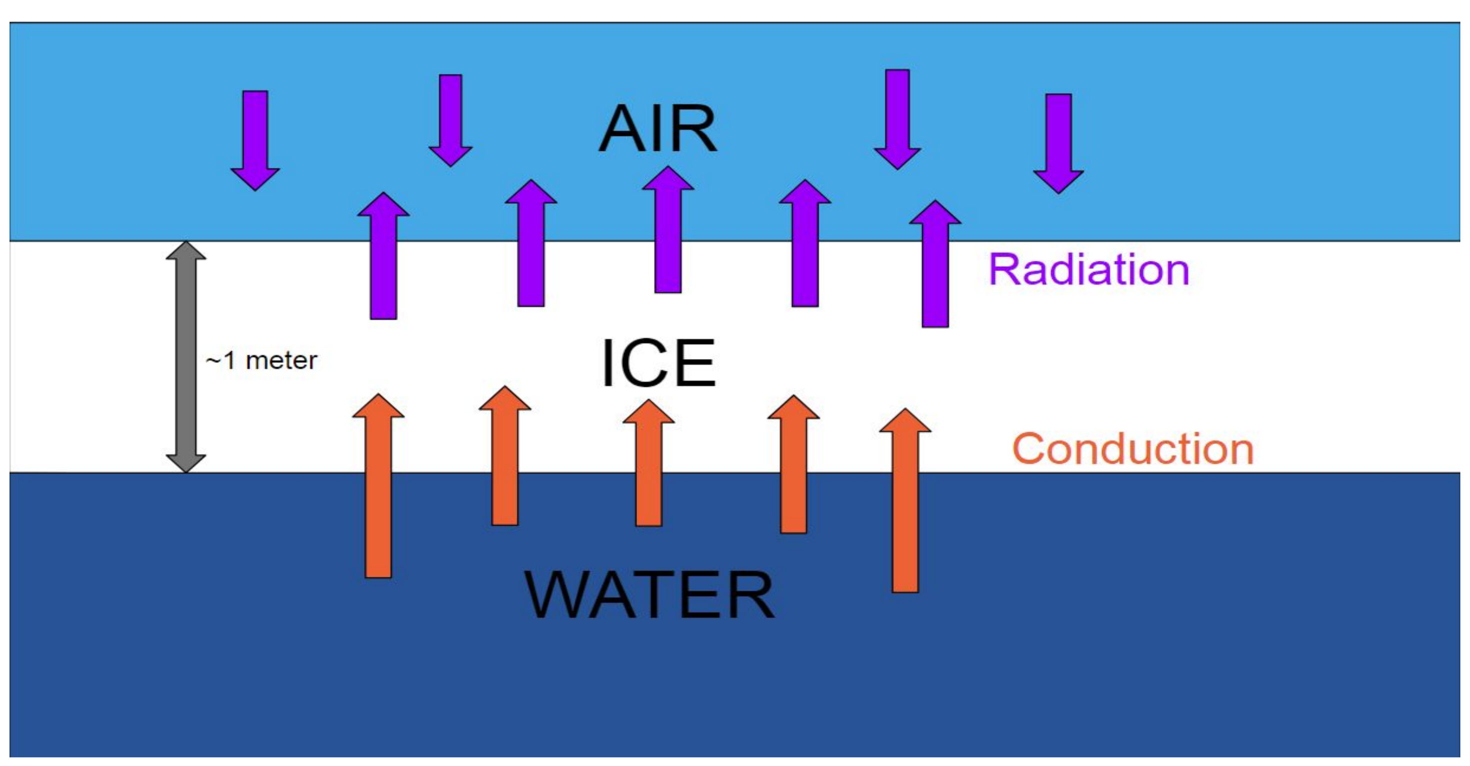

Figure 1: This is a simplified visual of the sea ice and how its thermal properties relate to the seawater and the air. Ice conducts heat up from the sea water and then radiates most of that heat into the air. The air is also radiating heat back into the ice, but it is radiating less into the ice than the ice is radiating out because the air is colder than the ice surface.

Research Question

Does the temperature at the surface of the sea ice correlate with the thickness of the ice?

Research Goals

- Our goal was to design a mathematical model that predicted the temperature distribution throughout the sea ice
- Made difficult by the “slush layer,” because thermal conductivity varies greatly throughout this layer
- Ideally, the final product would predict the depth of the ice using the surface temperature of the ice and the known temperature of seawater (-1.9° Celsius)

Modeling Energy Transfer

- Ice conducts and radiates thermal energy, therefore both had to be taken into consideration when designing this model
- Ice conducts heat from the seawater (higher temperature T) to the surface of the ice (lower T)
  - Conduction equation: $P/A = k\Delta T/L = k(T_f - T_i)/L$ ~ Watts/m^2
- All things radiate heat energy. Just as shown in Figure 1, ice is both radiating thermal energy and taking in thermal energy that is radiating from the air
  - Radiation equation: $P/A = e\sigma T^4$ ~ Watts/m^2
- Emissivity is another factor in our model, and it is a value between 0 and 1 that represents the efficiency of heat radiating into or out of a substance
  - The emissivity of ice is e=0.98
  - This value is the same for heat radiating into as well as out of the ice

Radiation:	Variables:
$P/A = e\sigma(T_{surface}^4 - T_{air}^4)$	P: power e: emissivity σ: Stefan-Boltzmann constant A: area
Conduction:	T: temperature (Kelvin) k: thermal conductivity
$P/A = (k\Delta T)/\Delta z$	Δz: thickness of the material

Figure 2: These are the final two equations used in the model attempts. To the right is a key listing every variable shown in the equations and what they represent. Thermal conductivity and emissivity are known values, as well as the temperature of the seawater (-1.9°C). The temperatures of the surface of the ice and the air can be estimated or directly measured. Thickness can be measured as well, but the slush layer prevents the measured value from being able to properly factor in the fading of ice into water.

Method

- Our first modeling attempt was very basic, leaving k as a constant value of 2.45 Watts/(meter*Kelvin)
  - This resulted in what resembled a linear pattern
  - Used many assumptions in this model, making it less accurate
- Our second attempt modeled the k-value and used starting points (T and k of seawater and T and k of ice) and Excel to force values into an exponential curve
  - Model formula (given T): k=Log(T(Kelvin) / 315.47K) / (Log(e) * -0.098)
  - Accounts for varying conductivity of ice better than original model
- Our final model utilized Excel’s iterative function feature to create an even more accurate model
  - Iterative cells: $T_{middle}=(k_{below}*T_{below}+k_{above}*T_{above}) / (k_{below}+k_{above})$
  - Flux at surface: $T_{top\ layer}=(e\sigma\Delta z(T_{surface}^4-T_{air}^4)/k_{ice(not\ varied)})+T_{surface}$

Results

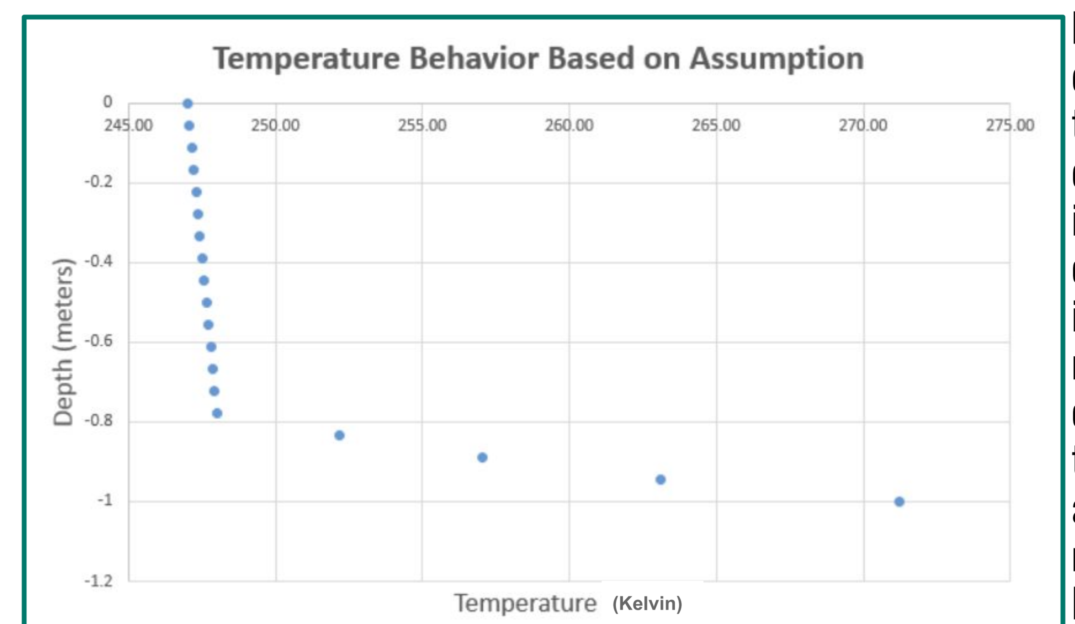

Figure 3: This graph depicts one model of temperature versus depth. Initially the curve is linear assuming a constant k while in solid ice (first 80 cm), then resembles an exponential curve for the last 20 cm as we allowed k to vary to represent the slush layer.

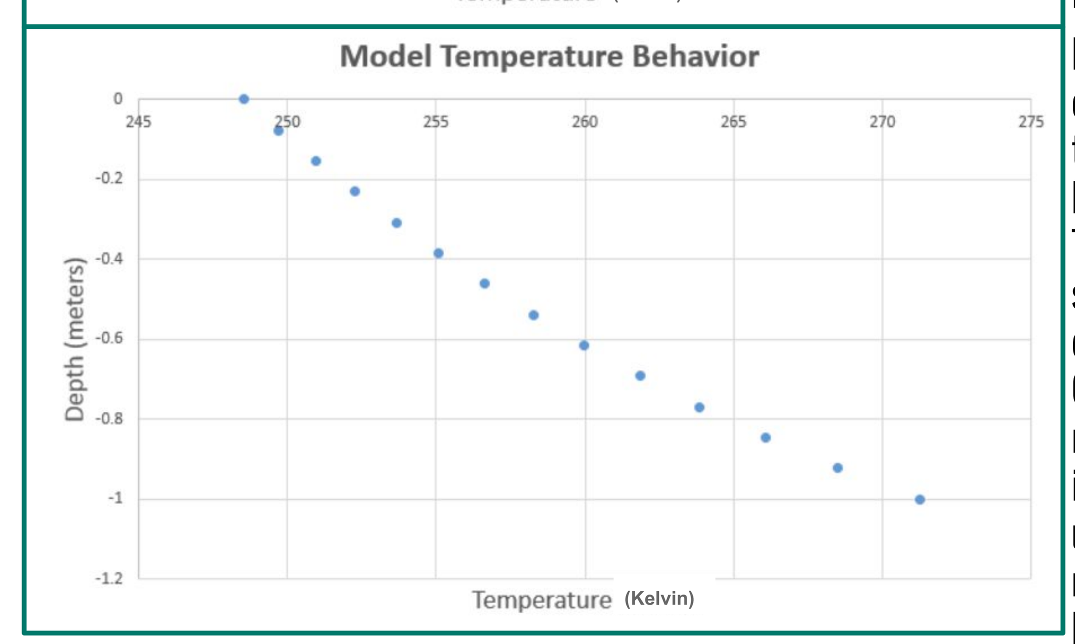

Figure 4: This is a graph of the 12 predicted temperature layers and the boundary temperatures. The boundary temp. of the surface is an average of data collected with the C.A.R.T.T. across a relatively flat expanse of ice. The k-value varies using the exponential model we created of the k-value through the slush.

Conclusion

- Model shows promise of developing a method for predicting ice depths based on surface temperatures
- More data and experimentation will lead to a better model

Acknowledgements

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